Rg for kt transition

Monday, July 12, 2021 10:32 PM

RG for O(n) model and KT transion (short summary).

For O(n) model

$$\mathcal{L}[\phi(\vec{r})] = \int d\vec{r} \left\{ \frac{t}{2}(\bar{\phi}\cdot\bar{\phi})^2 + \frac{1}{2}(\partial_i \phi_\alpha \partial_i \phi_\alpha) + u(\bar{\phi}\cdot\bar{\phi})^2 + \dots \right\}$$

where $\phi = \{\phi_1, \phi_2, \dots, \phi_n\}$ is an n-component unit vector.

Mean field answer for critical exponents is independent of n, which breaks down for $d<4$.
A similar ϵ-expansion can be done for $d<4$.
Corresponding RG flow

$$\frac{dt}{dr} = 2t + 4u(n+2)\frac{S_d}{(2\pi)^d}\cdot\frac{\Lambda^d}{t+\Lambda^2} - Au^2$$

$$\frac{du}{dt} = (4-d)u - \frac{4(n+8)S_d}{(2\pi)^d}\cdot\frac{\Lambda^d}{(t+\Lambda^2)^2}u^2$$

Note that the zeroth order terms do not depend on n, which is consistent with meanfield results.

For upto linear order in ϵ, the critical exponents depend on n and correspond to different WF fixed point.

$$t^* = -\frac{n+2}{2(n+8)}\cdot\Lambda^2\cdot\epsilon \quad \text{and} \quad u^* = \frac{2\pi^2}{n+8}\epsilon$$

[see Kardar Vol 2 for more details]

KT-transition

XY-model in 2d:

$$H = -J\sum_{\langle ij\rangle} \bar{\sigma}_i\cdot\bar{\sigma}_j$$

[Ref: Kardar Vol.2]

In continuum limit

$$\mathcal{L} = \frac{J}{2}\int d\bar{r}\,(\nabla\phi)^2 + \underbrace{2JN}_{\text{Energy of completely aligned state.}}$$

In 2d it does not have longrange order due to Mermin-Wagner theorem. Yet there is a transition mediated by topological defects separating phases with power-law correlation and exponential correlation. The transition is due to vortex binding-unbinding. Vortices interact by an effective coulomb interactions in 2d and the transition is effectively described by that of 2d neutral Coulomb gas.

$$\rightarrow H_n = -K\sum_{i\neq j}^{n} q_i q_j \log\left|\frac{r_i - r_j}{r_0}\right| + nE_c$$

r_0 is the scale where this effective theory is defined.

q_i = winding number of a vortex.

In grand canonical ensemble

$$\rightarrow Z = \mathrm{Tr}\left(e^{-H_n}\cdot z^n\right)$$

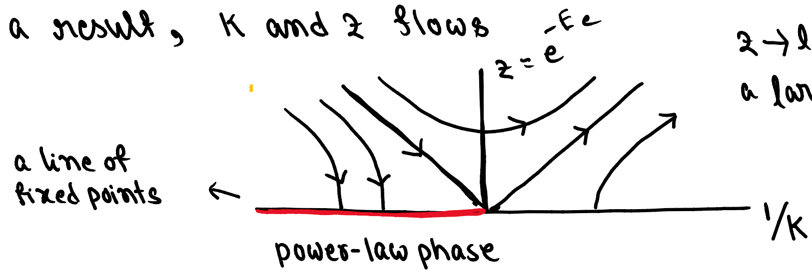
here $z = e^{-E_c}$ is vortex fugacity
where E_c is the core energy of vortex

$$\rightarrow Z_n = \mathrm{Tr}\left(e^{-H_n} \cdot z^n\right)$$

here $z = e^{-E_c}$ is vortex fugacity
where E_c is the core energy of vortex.

The RG is performed by coarsegraining over vortex-anti-vortex pairs as the scale r_0 is changed.

As a result, K and z flows



$z \to$ large indicates a large number of free vortices

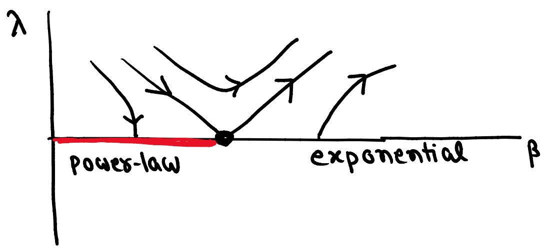
$z = 0$ indicates there are no vortices at large-scale
They are present as bound pairs in short distances.

Other models relevant to coulomb gas is sine-Gordon model

$$\mathcal{L} = \int d\vec{r}\left[\frac{1}{2}(\nabla\phi)^2 - \lambda\cos(\beta\phi)\right]$$

where ϕ is real scalar field.

Corresponding RG-flow



$$\frac{d\lambda}{dr} = \left(2 - \frac{\beta^2}{4\pi}\right)\lambda$$

$$\frac{d\beta}{dr} = -c \cdot \beta^3 \lambda^2$$